GRAIL
an omni-directional gravitational-wave detector

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Abstract. An cryogenic spherical and omni-directional resonant-mass detector proposed by the GRAIL collaboration is described.

Spherical resonant-mass detectors have been discussed in the literature since the early 70's [1, 2, 3]. They have a number of obvious advantages over Weber bars:

- Spherical detectors have the largest mass for a given linear dimension.
- They are always optimally oriented with respect to any source.
- They can determine the transverse plane of polarization of a signal, allowing the reconstruction of the direction of the source (modulo a reflection in the transverse plane).
- They are sensitive to a scalar component of gravitational radiation.

Moreover, in view of recent developments in the design of electro-mechanical transducers the fractional bandwidth of a spherical detector, equipped with a sufficient number of such transducers, is expected to reach values of the order of 20%, an improvement by two orders of magnitude over resonant detectors presently in operation. The development of a spherical resonant-mass detector is therefore considered an attractive new step towards the goal of observing gravitational radiation from astrophysical sources [4, 5, 6].

Recently we presented a preliminary design for a 115 ton spherical detector, to be made out of a high-Q alloy such as Cu-Al ($Q \geq 10^7$), and operated at a temperature in the range 10-20 mK [7]. This is to be achieved by integrating a high-power ^{3}He-^{4}He dilution refrigerator into the system.

The free vibration modes of a perfect sphere can be computed analytically [8, 9, 2, 10]. In general the displacement field $\vec{u}(\boldsymbol{r}, t)$ can be expanded as

$$\vec{u}(\boldsymbol{r}, t) = \sum_k a_k(t) \vec{\psi}_k(\boldsymbol{r}), \tag{1}$$

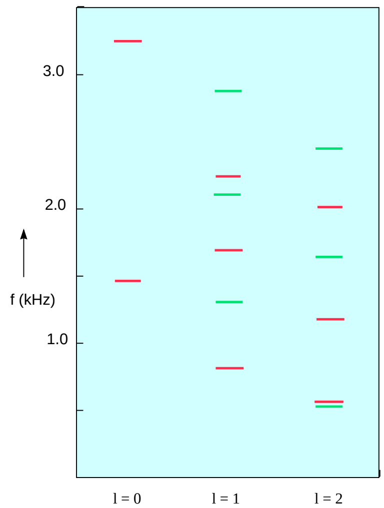

Fig.1 Red: spheroidal mode; green: toroidal modes

where k collectively denotes the characteristic numbers (n, l, m) of the free vibrations, with n the number of zero's of the radial component and (l, m) specifying the angular dependence in terms of the 2^l-multipole character. In the absence of dissipation, the time dependence of the modes is described by the oscillator equation

$$\ddot{a}_k + \omega_k^2 a_k = 0, \tag{2}$$

with ω_k the angular frequency of the free vibration mode. Depending on the reflection parity, modes may be classified as spheroidal or toroidal; only the spheroidal modes couple to gravitational waves. The frequencies $f_k = \omega_k/2\pi$ are plotted for the lowest spheroidal and toroidal modes of various multipole character in fig.1. Clearly, the fundamental mode, which for the GRAIL sphere has a frequency close to 700 Hz, is of quadrupole type, thus matching the tensor character of the gravitational field.

In the presence of external forces, eq.(2) for the amplitudes is replaced by the inhomogenous equation

$$\ddot{a}_k + \omega_k^2 a_k = f_k(t), \tag{3}$$

where the driving term $f_k(t)$ is given in terms of the force density $\vec{f}(\boldsymbol{r}, t)$ by

$$f_k(t) = \frac{1}{M} \int d^3r \, \vec{\psi}_k \cdot \vec{f}, \tag{4}$$

with M the total mass of the sphere. A weak gravitational wave is described by a force density of quadrupole type, i.e. $l = 2$; to the amplitude a_m of the mth quadrupole mode it imparts an acceleration

$$f_m(t) = \frac{1}{2}\ddot{h}_m(t)\chi R, \qquad m = -2, -1, ..., +2. \tag{5}$$

The dimensionless constant of proportionality χ depends weakly on the Poisson ratio σ of the material; its value $\chi \approx 0.6$ varies only by a few percent over a large range of values of σ. For the GRAIL detector the radius $R = 1.5$ m.

For example, an exponentially decaying burst of gravitational radiation with the time variation of its amplitude of the form

$$\dot{h}_m(t) = \frac{h_m}{\tau}\theta(t)e^{-t/\tau}, \tag{6}$$

produces for times $t \gg 2\tau$ an amplitude

$$a_m(t) = a_m^{(0)}(t) + \frac{h_m}{1 + \omega_0^2\tau^2}\left(\cos\omega_0 t + \omega_0\tau\sin\omega_0 t\right). \tag{7}$$

Here ω_0 is the frequency of the free quadrupole mode, and $a_m^{(0)}(t)$ is an arbitrary free vibration of the system, providing a background against which the signal is to be measured. If the source is at a large distance d, and converts an energy $E_{burst} = \eta M_\odot c^2$ into an isotropic exponential burst of gravitational radiation with amplitude decay time τ, then a simple calculation shows that typically

$$h_m = \frac{2}{d}\sqrt{\eta R_\odot c\tau} = 0.6 \times 10^{-13}\frac{\sqrt{\eta\tau}}{(d/1 \text{ kpc})}. \tag{8}$$

with $R_\odot \approx 3$ km the Schwarzschild radius of the sun. For a 1 ms burst at $d = 10$ Mpc converting 0.1 percent of a solar mass into gravitational waves the contribution to the amplitude a_m is of the order $h_m/\omega_0\tau \approx 10^{-21}$.

Under realistic conditions, the detected signal will be accompanied by external background (e.g., seismic, electromagnetic, cosmics) and internal noise. The GRAIL design incorporates a chain of masses and rods to attenuate (by reflection) external vibrations by more than 300 dB. This should be sufficient to eliminate high-frequency seismic background. A point of special concern will be to avoid short-circuiting this attenuation system by the dilution refrigerator cooling the sphere via the last masses in the chain. Finally, cosmics can be largely eliminated by placing the detector underground.

Important sources of noise in the detector are the thermal noise of the sphere, and the displacement and force noise of the amplifiers. As illustrated in fig.2, the latter are conventionally described by a noise temperature T_n and a noise impedance r_n, defined by:

$$kT_n = \sqrt{S_u S_f}, \qquad r_n = \sqrt{S_f/S_n}. \tag{9}$$

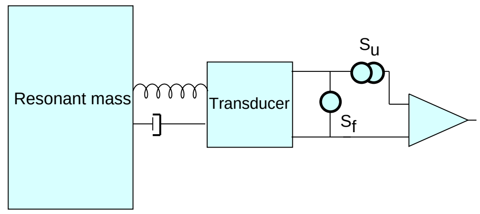

Fig.2 Noise model of resonant-mass detector with transducer

To optimize the signal-to-noise ratio, one should match the noise impedance with the output impedance of the tranducer, and the noise temperature with the effective temperature of the sphere, $T_{eff} = T/\beta Q$, with β the electromechanical coupling between the sphere and the amplifier [11]. In the optimal case the detector is quantum-limited, with $kT_n = hf_0 \approx 0.4 \times 10^{-30}$ J. To achieve this, a high Q-value is indispensible: $\beta Q > 10^6$, whilst the amplifier also is to approach the quantum limit.

Using the methods described in [5, 12], we have computed the sensitivity of the GRAIL detector equipped with six double mode transducers in a TIGA-configuration. A typical result for the noise of a quantum-limited detector, for three values of the noise impedance: $r_n = (100; 1000; 10,000)$ N.s/m refered back to the equivalent gravitational wave input, is sketched in fig. 3. For the optimal value $r_n = 1000$ the detector has a noise less than the equivalent strain of $10^{-23}/\sqrt{\mathrm{Hz}}$ over a range of almost 100 Hz in frequency.

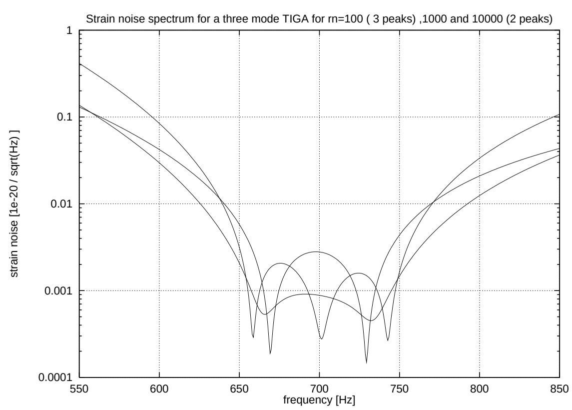

Fig.3 Sensitivity of quantum limited GRAIL sphere with double-mode transducers

The sensitivity of a quantum-limited GRAIL is compared to that expected for the LIGO-interferometer in its advanced phase [13] in fig. 4. From this figure

it may be infered, that the sensitivities of the two types of detectors are comparable. In contrast, in characteristics in frequency range, and in band width vs. directional sensitivity, they will be largely complementary. As such, the GRAIL detector might become an important element in a worldwide network of gravitational wave detectors.

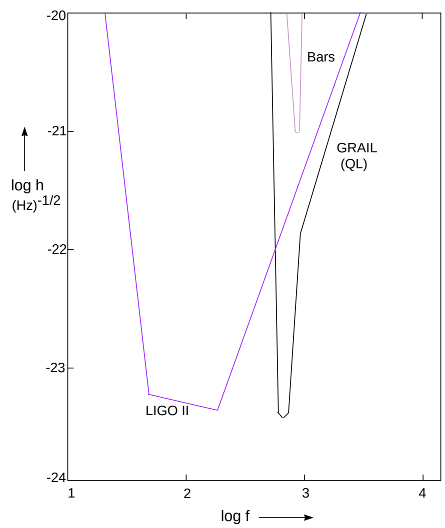

Fig. 4 Projected sensitivity of quantum-limited GRAIL sphere compared to advanced LIGO and cryogenic Weber bars

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